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Large-Scale Stochastic Linear Programs: Importance Sampling and Benders Decomposition

by

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Large-Scale Stochastic Linear Programs — Importance Sampling and Benders Decomposition*

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Abstract

The paper demonstrates how large-scale stochastic linear programs with recourse can be efficiently solved by using a blending of classical Benders decomposition with a relatively new technique called importance sampling. Numerical results of large-scale problems in the area of expansion planning of power systems and financial planning are presented.

1. Introduction

Solutions obtained from deterministic planning models are usually unsatisfactory because they fail to hedge against unfavorable events which may occur in the future. Stochastic models address this shortcoming, but in the past have seemed to be untractable because, even for a relatively small number of parameters, subject to uncertainty the size of such problems can get very large. Stochastic problems have been studied extensively in the literature since Dantzig (1955), for example Birge (1985), Ermoliev (1983), Frauendorfer (1988), Higle and Sen (1989), Kall

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(1979), Pereira et al. (1989), Rockafellar and Wets (1989), Ruszczynski (1986), Wets (1984) and others contributed in this area (Ermoliev and Wets (1988)). The paper addresses a common class of stochastic models, namely the stochastic linear programs with recourse. We discuss how decomposition techniques and importance sampling can be applied to solve them (Dantzig and Glynn (1990), Infanger (1990)).

2. A Class of Multi-Stage Stochastic Linear Programs

Large-scale deterministic mathematical programs, used for operations and strategic planning, often are dynamic linear programs. These problems have a staircase (multistage) matrix structure. In general, the size of these stochastic problems can get extremly large because the number of scenarios grows exponentially with the number of periods. We will, however, address a certain restricted class whose number of scenarios grows linearly with the number of stages: The problem (whose constraints are stated below) breaks down into two parts: a deterministic dynamic part and a stochastic part. We call the deterministic part the master-problem. It is a dynamic linear program with T stages. The vectors c_t and b_t, and the matrices B_{t-1} and A_t are assumed to be known with certainty.

$$
\begin{array}{rrrl}
\multicolumn{4}{c}{\min \sum_{t=1}^{T} c_t\, x_t + \sum_{t=1}^{T} E(f_t y_t^{\omega_t})} \\
-B_{t-1}x_{t-1} + A_t\, x_t & & & = b_t, \;\; t = 1,\ldots,T,\; B_0 = 0 \\
-F_t^{\omega_t}\, x_t & & +D_t y_t^{\omega_t} & = d_t^{\omega_t},\, t = 1,\ldots,T,\; \omega_t \in \Omega_t \\
x_t, & & y_t^{\omega_t} & \geq 0
\end{array}
$$

Each stage is associated with a stochastic sub-problem. Uncertainty appears in the recourse-matrix $F_t^{\omega_t}$ and in the right hand side vector $d_t^{\omega_t}$ where ω_t, denotes an outcome of the stochastic parameters in period t, with Ω_t denoting the set of all possible outcomes in period t. The sub-problems in each stage are assumed to be stochastically independent. The sub-problem costs f_t and the technology matrix D_t are assumed to be deterministic parameters.

Facility expansion planning is an example of this type of formulation. The master-problem models the expansion of the facilities over time. Decision variables are the capacity built and the capacity available at time t. The sub-problems model the operation of these capacities in an uncertain environment. Take for example the case of expansion planning of power systems: The expansion or replacement of capacities of generators and transmission lines are determined in the master problem. The capacities at each period t are made available to the system for operation. The subproblems model the power system operation, the optimal sceduling of the available capacities to meet the demand for electricity. The availabilities of generators and transmission lines and the demands are uncertain and not known at the time when the expansion decision is made.

The approach is primarily "here and now" (Dantzig and Madansky(1961)) and justified by high investment costs and long lead-times for capacity expansion. However, as the operations subproblems are stochastically independent and only expected operation cost rather than the state of the system after period t affects the expansion plan (as failures of equipment get repaired, and uncertainty in the demands are interpreted as deviations from a demand path), "here and now" is equivalent to "wait and see". That means that the optimal decision in period $t+1$ depends only on the capital stock on hand at the start of period $t+1$ and is independent of any observed outcomes in period t, i.e. the same optimal capacity expansion decision would be made before as after period t operations. Thus the facility expansion plan can be laid out at the beginning for the whole planning horizon based on the expansion costs and the expected operation costs. This permits the multi-stage problem to be treated as if it were a two-stage problem. The first "stage" concerns the single decision of what facility expansion will be in all future periods without knowledge of the particular outcomes of the uncertainty parameters in future periods. The second "stage" concerns the operations problems, where the

recourse decisions made depend on the realizations of the stochastic parameters. Note that for $T = 1$, the problem is exactly a two stage stochastic linear program with recourse. For $T \geq 2$ the problem is a two "stage" problem with the second stage consisting of T independent subproblems.

3. Multidimensional Integration

The difficulty of solving large-scale stochastic problems arises from the need to compute multiple integrals or multiple sums. The expected value of the second stage costs in period t (we suppress the index t for this discussion), e.g. $z = E(fy^\omega) = E(C)$ is an expectation of functions $C(v^\omega), \omega \in \Omega$, where $C(v^\omega)$ is obtained by solving a linear problem. V (in general) is a h-dimensional random vector parameter, e.g. $V = (V_1, \ldots, V_h)$, with outcomes $v^\omega = (v_1, \ldots, v_h)^\omega$. For example V_i represents the percent of generators of type i down for repair or transmission lines not operating and v_i^ω the observed random percent outcome. The vector v^ω is also denoted by v, and $p(v^\omega)$ alias $p(v)$ denote the corresponding probability. Ω is the set of all possible random events and is constructed by crossing the sets of outcomes $\Omega_i, i = 1, \ldots, h$ as $\Omega = \Omega_1 \times \Omega_2 \times \cdots \times \Omega_h$. With P being the probability measure under the assumption of independence the integral $E\ C(V) = \int C(v^\omega) P(d\omega)$ takes the form of a multiple integral $E\ C(V) = \int \cdots \int C(v) p(v) dv_1 \ldots dv_h$, or, in case of discrete distributions, the form of a multiple sum $E\ C(V) = \sum_{v_1} \cdots \sum_{v_h} C(v) p_1(v_1) \ldots p_h(v_h)$.

In the following discussion we concentrate on discrete distributions. This is not a restriction as the approach can be easily adapted for continuous distributions. In practical applications all distributions can be approximated with sufficiant accuracy by discrete ones. Even for h as small as 20 the number of terms in the multiple sum computation gets easily out of hand and the problem is no longer practical to solve by direct summation. This is especially true because function avaluations are computationally expensive since each term in the multiple sum requires the solution

of a linear program.

4. Importance Sampling

Monte Carlo Methods are recommended to compute multiple integrals or multiple sums for higher h-dimensional sample spaces (Davis and Rabinowitz (1984), Glynn and Iglehart (1989)). Suppose $C^\omega = C(v^\omega)$ are independent random variates of $v^\omega, \omega = 1, \ldots, n$ with expectation z, where n is the sample size. An unbiased estimator of z with variance $\sigma_{\bar{z}}^2 = \sigma^2/n$, $\sigma^2 = var(C(V))$ is

$$\bar{z} = (1/n) \sum_{\omega=1}^{n} C^\omega.$$

Note that the standard error decreases with $n^{-0.5}$ and the convergence rate of $\bar{z}$ to z is independent of the dimension of the sample space h. We rewrite $z = \sum_{\omega \in \Omega} C(v^\omega) p(v^\omega)$ as

$$\sum_{\omega \in \Omega} \frac{C(v^\omega) p(v^\omega) q(v^\omega)}{q(v^\omega)}$$

by introducing a new probability mass function $q(v^\omega)$ and we obtain a new estimator of z

$$\bar{z} = \frac{1}{n} \sum_{\omega=1}^{n} \frac{C(v^\omega) p(v^\omega)}{q(v^\omega)}$$

by sampling from $q(v^\omega)$. The variance of $\bar{z}$ is given by

$$var(\bar{z}) = \frac{1}{n} \sum_{\omega \in \Omega} \left(\frac{C(v^\omega) p(v^\omega)}{q(v^\omega)} - z \right)^2 q(v^\omega).$$

Chosing $q^*(v^\omega) = \frac{C(v^\omega) p(v^\omega)}{\sum_{\omega \in \Omega} C(v^\omega) p(v^\omega)}$ would lead to $var(\bar{z}) = 0$, which means one could get a perfect estimate of the multiple sum from only one estimation. Practically however, this is useless since to compute $q(v^\omega)$ we have to know $z = \sum_{\omega \in \Omega} C^\omega p(v^\omega)$, which we eventually want to compute. The result however helps to derive a heuristic for choosing q. It should be proportional to the product $C(v^\omega) p(v^\omega)$ and should have a form that can be integrated easily. Thus a function $\Gamma(v^\omega) \approx C(v^\omega)$ is sought,

which can be integrated with less costs than $C(v^\omega)$. Additive and multiplicative (in the components of the stochastic vector v) approximation functions and combinations of these are potential candidates for our approximations. In particular, we have been getting good results using $C(V) \approx \sum_{i=1}^{h} C_i(V_i)$. We compute q as

$$q(v^\omega) \approx \frac{C(v^\omega)p(v^\omega)}{\sum_{i=1}^{h}\sum_{\omega\in\Omega_i} C_i(v^\omega)}.$$

In this case one has to compute only h 1-dimensional sums instead of 1 h-dimensional sum. The variance reduction depends on how well the approximation function fits the original cost function. If the original cost function has the property of additivity (seperability) the multiple sum can be computed exactly by h 1-dimensional sums. If the addidive model is a bad approximation of the cost function the only "price" that has to be paid is increasing the size of samples. If the observed variance is too high using a starting sample size the size of sample is adjusted higher. Actually we use a variant of the addidive approximation function. By introducing $C(\tau)$, the costs of a base case, we make the model more sensitive to the impact of the stochastic parameters v.

$$\Gamma(V) = C(\tau) + \sum_{i=1}^{h} C(\tau_1, \ldots, \tau_{i-1}, V_i, \tau_{i+1}, \ldots, \tau_h) - C(\tau)$$

We denote this as a marginal cost model. τ can be any arbitrary chosen point of the set of values v_i, $i = 1, \ldots, h$. For example we choose τ_i as that outcome of V_i which leads to the lowest costs, ceteris paribus.

5. Benders Decomposition

We decompose the 2-stage multi-period stochastic linear problem by applying Benders (1962) decomposition. (Van Slyke, Wets (1969)):

The master problem:

$$
\begin{aligned}
z_M^L = \min & \sum_{t=1}^{T} c_t\, x_t + \sum_{t=1}^{T} \theta_t \\
-B_{t-1}x_{t-1} + A_t\, x_t & = b_t, \quad t = 1,\ldots,T, B_0 = 0 \\
-G_t^l\, x_t + \alpha_t^l \theta_t & \geq g_t^l, \quad t = 1,\ldots,T, l = 1,\ldots,L \\
x_t & \geq 0
\end{aligned}
$$

where the latter constraints, called cuts, are initially absent but are inserted in later iterations. The master problem is optimized to obtain an approximate optimal feasible solution $x_t = \hat{x}_t^l$ that is used as input to the subproblems.

The sub-problems for ω_t in period t:

$$
\begin{aligned}
& z_t^{\omega_t}(\hat{x}_t^l) = \min f_t y_t^{\omega_t} \\
& \pi_t^{\omega_t}(\hat{x}_t^l): \quad D_t y_t^{\omega_t} = d_t^{\omega_t} + F_t^{\omega_t}\hat{x}_t^l, \quad \omega_t \in \Omega_t, t = 1,\ldots,T \\
& \qquad\qquad y_t^{\omega_t} \geq 0, \;\; \hat{x}_t^l \; given,
\end{aligned}
$$

where $\pi_t^{\omega_t} = \pi_t^{\omega_t}(\hat{x}_t^l)$ are dual multipliers corresponding to the constraints and $z_t^{\omega_t} = z_t^{\omega_t}(\hat{x}_t^l)$ is the value of the objective as functions of $(\hat{x}_t^l)$. These are used to generate the next cut for the master.

The cuts: for $t = 1, 2, \ldots, T$,

$$
G_t^l = E(\pi^{\omega_t} B^{\omega_t}), \quad g_t^l = E(\pi^{\omega_t} d^{\omega_t}), \quad z_t(\hat{x}_t^l) = E(z_t^{\omega_t}), \quad \pi_t^{\omega_t} = \pi_t^{\omega_t}(\hat{x}_t^l).
$$

Lower (LB^L) and upper (UB^L) bounds to the problem:

$$
LB^L = z_M^L, \quad UB^L = \min\{UB^{L-1}, \sum_{t=1}^{T}(c_t\hat{x}_t^l + z_t(\hat{x}_t^l))\}, UB^0 = \infty
$$

$\hat{x}_t^l$ is the optimal solution of the master problem in iteration l, $\pi_t^{\omega_t}(\hat{x}_t^l)$ is the optimal dual solution of subproblem ω_t, given $\hat{x}_t^l$. Note that if the subproblems are infeasible, a slightly different definition of the cut is used. $\alpha = 0$ corresponds to feasibility cuts and $\alpha = 1$ to optimality cuts. Solving the master problem in iteration l we obtain a trial solution $\hat{x}_t^l$ which we pass to the subproblems. By solving a

sample of sub-problems $\omega_t, \omega_t \in N_t, t = 1,\ldots,T$, according to the importance sampling scheme we compute estimates of the second stage costs z_t and estimates of the gradients G_t^l and the right hand sides g_t^l of the cuts. Cuts represent an outer linearization of the second stage costs expressed in first stage decision variables and θ_t. Note that there is one cut for each period t. The cuts are added to the master problem and the master problem is solved again. The objective function value of the master problem gives a lower bound estimate and the total expected costs of a trial solution $\hat{x}_t^l, t = 1,\ldots,T$ gives an upper bound estimate to the objective function value of the problem. If the lower and the upper bound are sufficiently close, which is tested by a Student-t test, the problem is considered to be solved. Lower and upper bounds can be seen as a sum of *i.i.d.* random terms which for sample sizes of 30 or more can be assumed normally distributed with known (derived from the estimation process) variances. A 95% confidence intervall of the optimal solution is computed. See Dantzig and Glynn(1990) and Infanger(1990) for details of the algorithm.

6. Numerical Results

This method for solving large-scale two-stage stochastic linear programs with recourse has been implemented. The code of MINOS (Murtagh and Saunders (1983)) has been adapted for this purpose as a subroutine for solving both the master-problem and the sub-problems. When solving large numbers of sub-problems it is important for the performance of the algorithm to take advantage of good starting bases. Computation time can be reduced dramatically by solving **first** an expected value problem by replacing the stochastic parameters by their expectations. The expected value solution of the resulting deterministic problem is then used as a starting point for the stochastic solution. Additionally we keep cuts obtained from the expected value problem to initially guide the algorithm. It can be shown that

cuts obtained from the expected value problem are valid for the stochastic problem. They are "weak" and get repaced as the algorithm proceeds. The code uses sparce matrix techniques and efficient data structures for handling large-scale problems.

Computational results of the large scale test problems are represented in Table 1. Besides the solution of the stochastic problems, the results from solving the expected value problems are also reported. We also report on the estimated expected costs if the expected value solution is used as the decision in a stochastic environment. The objective function value of the true stochastic solution has to lie between the minimum value of objective function of the deterministic problem and the expected costs of the expected value solution.

Expansion planning of multi-area power systems

WRPM is a multi-area capacity expansion planning problem for the western USA and Canada. The model is very detailed and covers 6 regions, 3 demand blocks, 2 seasons, and several kinds of generation and transmission technologies. The objective is to determine optimum discounted least cost levels of generation and transmission facilities for each region of the system over time. The model minimizes the total discounted costs of supplying electricity (investment and operating costs) to meet the exogenously given demand subject to expansion and operating constraints. A description of the model can be found in Dantzig et. al. (1989). In the stochastic version of the model the availabilities of generators and transmission lines and demands are subject to uncertainty. There are 13 stochastic parameters per time period (8 stochastic availabilities of generators and transmission lines and 5 uncertain demands) with discrete distributions with 3 or 4 outcomes. The model covers a time horizon of 3 future periods of 10 years each. Thus the total number of stochastic parameters is 39. The operating sub-problems of each period are stochastically independent. The number of universe scenarios is larger than $5 \cdot 10^6$ per period. In the deterministic equivalent formulation the problem if it were

possible to state it would have more than 4.5 billion constraints.

The stochastic WRPM is solved by using a sample size of 100 per period. It takes 129 iterations to obtain the expected value solution and additional 68 iterations to compute the stochastic solution. The objective function value of the stochastic solution was estimated as 199017.4 with an amazingly small 95% confidence intervall of 0.029% on the left side and 0.067% on the right side. Thus the optimal solution lies with 95% confidence between $198959.3 \leq z^* \leq 199164.1$. The expected costs of the expected value solution (202590.3) and the objective function value of the stochastic solution differ significantly from the expected costs of the optimal stochastic solution. The problem was solved in 687 minutes on a Toshiba T5200 laptop personal computer. This time includes time to solve 26295 linear sub-problems of the size of 302 rows and 289 columns and 197 master problems.

Multi-period Portfolio Management

LP42 is a portfolio management test-problem, formulated as a network problem. It is a modified version of test-problems found in Mulvey and Vladimirou (1989). The problem is to select a portfolio which maximizes expected returns in future periods taking into account the possibility of revising the portfolio in each period. There are also transaction costs and bounds on the holdings and turnovers. The test problem covers a planning horizon of four future periods. The returns of the stocks in the four future periods are assumed to be independent stochastic parameters, discretely distributed with 3 outcomes each; this formulation differs from that of Mulvey and Vladimirou who restricted the problem size by looking at a certain number of preselected scenarios. Like in Mulvey and Vladimirou the multi-period problem is viewed as a 2-stage problem, where all future periods are included in the second stage. With 13 stocks with uncertain returns, the problem has 52 stochastic parameters. The universe number of scenarios $6 \cdot 10^{24}$ is very large, so that the deterministic equivalent formulation of the problem if expressed explicitely

would have more than $1.9 \cdot 10^{27}$ rows. Here, the stochastic parameters appear in the B-matrix as well as in the D-matrix. In this case cuts from the expected value problem are not valid for the stochastic problem. The expected value problem and the stochastic problem are solved seperately. A sample size of 600 was chosen. The solution (objective function value 2.329) is obtained in 4 iterations. The 95% confidence intervall is very small given the large number of stochastic parameters, namely 0.536% on the left side and 0.767% on the right side. Thus with 95% confidence the objective function value of the optimal solution lies within $2.316 \leq z^* \leq 2.347$. The expected costs of the expected value solution is significantly different from the expected costs of the stochastic solution.

7. Conclusion

We have demonstrated that our approach using Benders decomposition and importance sampling is capable of solving large-scale problems of planning under uncertainty. Numerical results of large problems with numerous stochastic parameters indicate that very accurate solutions of the problems can be obtained using only small sample sizes.

Table 1: Large-scale problems: computational results

		WRPM	LP42
# periods		3	1
# iter stoch. (exp. val.)		197 (129)	4 (6)
sample size (per period)		100	600
exp. val. solution, obj		196471.4	1.611
exp. val. solution, exp. cost		202590.3	2.334
stochastic solution, obj		199017.4	2.329
est.	conf. left %	0.0292	0.536
	conf. right %	0.067	0.767
solution time (min)		687	209
Problem Size			
Master	rows	128	49
	columns	226	83
	nonzeros	413	133
Sub	rows	302	178
	columns	289	309
	nonzeros	866	570
# stoch. parameters (total)		39	52
# univ. scen. (per period)		$5 \cdot 10^6$	$6 \cdot 10^{24}$

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